AN ECONOMIC APPROACH TO VOLUNTARY ASSOCIATION

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Abstract

We develop an economic model of association based on voluntary contributions. Different equilibria corresponding to the different modes of formation of associations are analyzed and the results are compared with existing empirical literature. The main contribution consists in formalizing the voluntary association as a means of providing collective-consumption goods or services. We introduce the concept of the subjective quality as a possible incentive for volunteering. The model stresses the importance of non-pecuniary rewards and of accepted differentiation for well-functioning of voluntary organizations.

Key words: Voluntary Association, Public Good, Volunteering.


1 Introduction

The voluntary sector is considered as an extra-governmental provider of collective consumption goods representing an alternative for unsatisfied demanders (Weis-
brod, 1977, 1986.) It can be distinguished from the public and the private sectors by its means of providing goods and services. The public provision of goods or services is financed by compulsory taxes, while on the market individuals have to pay their consumption. With voluntary provision, on the other hand, individuals can choose to contribute to the provision of a good or service, whether they consume it or not (Sugden, 1984.) In this paper, we investigate the aspect of voluntary nonprofit organizations that involves individual commitment to the voluntary association.

Voluntary monetary contributions and voluntary work constitute important (but not the unique) resources for the functioning of associations. According to Havens et al. (2006), in the United States, about 90 percent of American households donate to nonprofits. In 2001, individual contributions accounted for 13.7 percent of the total resources of American nonprofits, while public support represented 9.9 percent (Tax Foundation, 2005.) Voluntary contributions are not necessarily monetary. They can also be represented by contributions in effort or time. In 1993, for instance, the value of voluntary work in the American nonprofits was estimated at about 182 billion dollars.

The example of the voluntary sector illustrates the fact that although voluntary contributions alone are often insufficient to make possible the provision of public goods, they cannot be ignored by economists. According to standard public good theory, the amount of the good provided through voluntary contributions will be sub-optimal because of free-riding. In theory, when agents are considered as pure altruists (Andreoni, 1989), i.e. when they are interested only in the total outcome of the collective action, public support leads to the complete crowding-out of voluntary contributions. However, empirical studies show that voluntary contributions are not completely crowded-out by public financial support. Moreover, in experimental studies, complete free-riding is not observed. Experimental studies allow
to distinguish between different determinants of voluntary contributions. Gener-
ally, these studies show that communication (for instance cheap-talk, negotiation
or promises) and the absence of anonymity tend to increase voluntary contributions
(Andreoni and Petrie, 2003.) Other social determinants, such as certain individ-
ual characteristics (Glaeser et al., 2000), cooperative values versus individualistic
values (Offermann et al., 1996), social environment (Carpenter et al., 2004) also
influence the provision of public goods. Clearly, the majority of experimental stud-
ies shed particular light on the influence of social factors in the provision of public
goods.

Empirical studies in the voluntary sector focus largely on the factors influencing
donors’ behavior. These factors include state support of the provision of public
goods, the total provision of public goods, the fund-raising expenditure of nonprof-
its and individual motivations. An interesting feature of the voluntary sector is that
governmental support of the public provision of public goods does not crowd-out
individual contributions. In some cases, one can even observe some crowding-in.
Okten and Weisbrod (2000) find no significant parameters of crowding-out in the
United States. Furthermore, they find that the area of higher education and research
presents some crowding-in effects (with parameters of 0.06 and 0.1 respectively).
In a study of international relief and development organizations, Ribar and Wilhelm
(2002) show that contributions to these organizations are only weakly affected by
public support, but they are positively influenced by fund-raising expenditure. In
experimental studies, the crowding-out is greater, but still incomplete. As Andreoni
suggested (1993), this is due to the intentional elimination of social factors in ex-
periments.

Another interesting feature is that voluntary organizations are recognized as being
capable of revealing demands and practising price discrimination, or personalized
prices (Hansmann, 1980; Ben-Ner, 1986.) The concept of Lindahl equilibria using
personalized prices shows that if such a price system is established, then efficiency can be attained in the provision of the public good. Ben-Ner (1986) pointed out that social relationships between the members, the constraint of non-distribution of benefits and the shared backgrounds of members all help nonprofits to establish personalized prices.

In this paper, we develop a model of voluntary association. Firstly, we consider a standard public good model where individuals can make voluntary monetary contributions. We derive some interesting properties concerning overcrowding, incentives, and the existence of different equilibria resulting from the different ways in which association can emerge.

Secondly, the concept of subjective quality is introduced into the basic model. The originality of the model is that we assume the public good to be characterized by at least two main components, namely the quantity and the quality. The quantity is considered here as a purely public component, insofar as all the members benefit equally from it. However, the quality of the public good is considered as a mixed (public and private) component. The agents can enjoy part of it in the same way, but there may exist certain characteristics of quality that are difficult or impossible to measure objectively. In a way, quality is always somewhat subjective, to the extent that perfect correspondence with the preferences of heterogeneous agents is unlikely to occur. In our model, the agents can contribute money or time and effort. The latter, or volunteering, allows them to influence the quality of the provided good (or service) according to their own preferences. The aim of the model is to formalize a set of assumptions about the functioning of voluntary organizations. The properties we derive from it help to understand better the recurring issues of nonprofit management and volunteering.

The paper is organized as follows. Section 2 presents the basic model and the conditions relating to overcrowding. Section 3 analyzes the individual and collective
incentives to make voluntary contributions. Section 4 shows the existence of different symmetric equilibria. Section 5 shows the existence of non-symmetric equilibria both without and then with budget constraints. In section 6 we introduce volunteering and the concept of subjective quality. Finally, section 7 concludes.

2 The Basic Model

The model presented in this section is an extension of a standard public good model. We consider a set of \( n \) agents \( i = 1 \ldots N \), with \( N > 1 \), who are members of an association. The number of members must be higher than one, by definition of an association. Each agent is endowed with an income \( w_i \), which can be used for private consumption and for the provision of a public good \( G \). All the members have to pay a compulsory amount \( c \), on top of which each member can add a voluntary pecuniary contribution \( d_i \) in order to increase production of the public good. At this stage, the utility function of each agent can be written as follows:

\[
U_{iG} = w_i - c - d_i + \frac{\theta}{N\gamma} (Nc + \sum_{j=1}^{N} d_j + X)^\alpha,
\]

\( 0 < \alpha < 1, \quad 0 \leq \gamma \leq 1, \)

The variable \( X \) has a double economic meaning. A positive value is interpreted as a fixed amount of monetary resources coming from exogenous origins (public grants, for instance.) A negative \( X \) describes the net amount of fixed costs (fixed costs less exogenous resources).

The parameters \( \alpha \) and \( \theta \) describe the production technology of the public good contributing to the individual utility, in the non-linear and linear forms respectively. The parameter \( \alpha \) can be interpreted in terms of more or less important decreas-
The parameter $\theta$ is always positive, because contributing to the good is assumed to generate a positive externality. The introduction of this parameter allows to vary the attractiveness of the good. In fact, as has been noted in some experimental studies, the attractiveness of a public good can positively influence individual voluntary contributions to it (Hichri, 2004.)

The parameter $\gamma$ denotes the publicness of the provided good by measuring its degree of rivalry (Bergstrom and Goodman, 1973; Blecha, 1987). A $\gamma$ tending towards zero corresponds to decreasing rivalry of the public good. At one extreme, $\gamma$ equal to zero describes the case of a collective-consumption good in the sense of Samuelson (1954). This good is completely non-rival and its consumption by one agent does not prevent others from consuming it. Think for instance of an environmental association campaigning against pollution, or an association fighting a disease. In these cases, each member benefits from the totality of the results of the collective action. On the contrary, a $\gamma$ equal to 1 describes a completely rival good, equivalent to a collectively provided private good equally shared among the members. As an illustration, this type of function may correspond to the running of housing cooperatives, fairly wide-spread nonprofit organizations in countries like Canada or Switzerland.

2.1 The Issue of Overcrowding

In our model, overcrowding may appear under some conditions. Actually, the share obtained by the agent $i$ will depend on $N$ and on the sharing rule characterized by $\gamma$. The overcrowding effects appear when the individual benefit for $i$ from the good provided diminishes as the number of members grows.

The case of $\gamma$ equal to zero implies the absence of overcrowding, insofar as it describes a pure public good (Samuelson, 1954). To understand overcrowding effects
when $\gamma$ is strictly positive, we study the variations of the production function (2) with the number of members $N$.

$$f(N) = \frac{1}{N^\gamma} (Nc + \sum_{j=1}^{N} d_j + X)^\alpha,$$  \hspace{1cm} (2)

**Proposition 1** In the absence of fixed costs (namely when $X \geq 0$) there is no overcrowding. More precisely:

- if $\gamma \geq \alpha$, association has no sense as the sharing rule cancels the marginal gain expected from any individual contribution.
- if $\gamma < \alpha$, there exists a threshold $N_1$ beyond which any additional contribution increases individual utility.

**Proposition 2** In the presence of fixed costs, (namely if $X < 0$) there can exist an overcrowding when $\gamma > \alpha$. More precisely:

- if $\gamma \leq \alpha$, there exists a threshold $N_0$ beyond which any additional individual contribution increases individual utility;
- if $\gamma > \alpha$ an association makes sense only for the population size $N \in [N_0, N_1]$.

Beyond the threshold $N_1$ there is overcrowding (see proof in Appendix A).

Proofs are presented in Appendix A. We begin by assuming symmetrical voluntary contributions. Then we study the sign of the production function with respect to $N$ in different configurations (fixed costs versus exogenous resources). To summarize, the sign of the derivatives of the production function varies as follows:

Table 1
The size of the association and overcrowding

<table>
<thead>
<tr>
<th>$N$</th>
<th>$N_0 = -\frac{X}{c}$</th>
<th>$N_1 = -\frac{\alpha}{(\gamma-\alpha)c^\alpha}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{df}{dN}$</td>
<td>Undefined</td>
<td>$&gt; 0$</td>
</tr>
<tr>
<td>Association</td>
<td>Overcrowding</td>
<td></td>
</tr>
</tbody>
</table>
At this stage, propositions 1 and 2 can be summarized in table 2.

<table>
<thead>
<tr>
<th>Cases</th>
<th>$X &gt; 0$</th>
<th>$X &lt; 0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\gamma &lt; \alpha$</td>
<td>No overcrowding</td>
<td>No overcrowding</td>
</tr>
<tr>
<td>$\gamma &gt; \alpha$</td>
<td>Association makes no sense</td>
<td>Overcrowding</td>
</tr>
</tbody>
</table>

Then we extend the scope of the results by considering the internal dynamics of the association. Olson (1971) has already stressed the role of the internal structure of associations, and in particular its impact on the group’s capacity to organize the provision of public goods. These internal structures can bring some formal or informal coordination to the process of providing the public good or organizing a collective action. An informal coalition can be created by individuals who decide to provide the public good even if they have to support the total cost, or at least a higher cost than the others. According to Olson, such coalitions can be formed in relatively large groups. In this case, the public good may be provided even in non-structured groups, as some participants may be able to provide the public good to the entire group.

In a first extension, we demonstrate that the results hold when $K$ first members, founders for instance, each make a fixed voluntary contribution while newcomers ($i > K$) only contribute the compulsory amount $c$.

Then we allow the possibility for $i > K$ members to make a positive voluntary contribution, by letting voluntary contributions vary for any $i \leq K$. We show that the general case results hold with these new extensions.
3 Incentives

For the rest of the paper we now consider $X \geq 0$ and $\gamma < \alpha$. The question we address now concerns individual and collective incentives in relation to the size of the association. This size can influence the incentives for agents to contribute individually or collectively, in a coordinated manner, via its effects on individual and collective returns. The attractiveness of the public good can depend on the number of persons sharing it. So what we are interested in here is the marginal return on the voluntary contribution of an $N^{th}$ newcomer. We compare the conditions of a positive return to those of a group of $N$ individuals who decided jointly to contribute. The positive return of a newcomer represents an ”individual incentive” to contribute, while the collective return constitutes a ”collective incentive” to form a coalition of ”volunteers”.

**Proposition 3** The individual incentive to contribute is at least $N$ times weaker than the collective incentive. The individual incentive assumes that:

$$\theta \alpha \geq N\gamma (Nc + \sum_{j=1}^{N-1} d_j + X)^{1-\alpha}. \tag{3}$$

Thus, it diminishes with $N$.

The collective incentive assumes that:

$$\theta \alpha \geq \frac{(Nc + X)^{1-\alpha}}{N^{1-\gamma}}. \tag{4}$$

Thus it increases with $N$ until

$$N \leq \frac{1 - \gamma X}{\gamma c} \tag{5}$$

and then it diminishes.
For proof see Appendix B.

**Example** As a simple illustration, let us take the case where \( X = 0 \) and \( \sum d_i = 0 \). By substituting this in (3) and (4), we obtain the conditions for the individual incentives:

\[
N \leq \frac{(\theta \alpha)^{\frac{1}{1-\alpha}}}{c},
\]

and for the collective incentives:

\[
N \geq \left( \frac{c^{1-\alpha}}{\theta \alpha} \right)^{\frac{1}{\alpha}}
\]

The figure below shows how the number of participants \( N \) varies with the amount of \( c \) due, for given parameters \( \theta \) and \( \alpha \).

Fig. 1. Illustration: \( N \) as a function of \( c \)

The curves of individual incentives presented in graphs (a) and (b) indicate the size \( N \) of the association, below which members have incentives to provide voluntary contributions individually. Comparison of the two graphs reveals the effect of the production technology of the public good on the incentives. The maximum size of the association below which the members have incentives to contribute individually is *ceteris paribus* higher when the production technology is less effective. Equiva-
lently, the size above which the members have incentives to form a coalition inside the association is *ceteris paribus* larger when the technological parameter is low.

**Corollary 3.1** To each size $N$ of voluntary association corresponds an optimal level of fee $c^*(N,X,\alpha,\gamma)$. In addition, this level increases with $N$, the number of members.

Proof is given in Appendix B.

### 4 Symmetric Equilibria

In this section, we consider the different equilibria corresponding to different modalities of formation of an association. Depending on these modes of emergence of association, the members can decide together to contribute an equal amount, or some members can form a coalition of volunteers. Some prior conditions may influence the process of formation of an association, for instance the heterogeneity in members’ incomes. Sociologists (Laville and Sainsaulieu, 1997) make a distinction between at least two main modes of formation of associations, depending on the position of the founders with regard to the action undertaken: - ”for self” and ”for others”. In the first case, the members self-organize with no distinction between a dominant category of active members and beneficiaries. In the organization ”for others”, on the contrary, there is a differentiation *a priori* between so-called ”weak” and ”strong” categories of agents (Laville and Sainsaulieu, 1997, p. 288.)

These modes of emergence affect the internal functioning of the organization. In associations whose members are not the direct beneficiaries, it is common to observe some detachment of ”weak” categories from the ”strong” actors. In this kind of organization, the mobilization of passive beneficiaries becomes an important is-
sue for management. In self-organized associations, on the contrary, the difficulty consists in reaching a consensus between the "strong actors" and pursuing the joint action in a sustained manner (Laville and Sainsaulieu, 1997, p. 290).

In the model proposed here, these situations are considered using the concepts of different equilibria. The N-symmetric equilibrium can be considered as the most socially desired result, in that it implies equal contributions from each member. It corresponds to the case of an association where all the participants belong to the same category of actors.

However, as Olson (1971) noted, an internal coalition can be formed, whose members make additional voluntary contributions. This situation can correspond to the case of the association "for others", with a distinction between "strong" and "weak" categories of participants. This kind of association is studied using the concept of K-symmetric equilibrium. Finally, introducing a budget constraint allows us to analyze the partial K-symmetric equilibrium. In this case, the members belonging to the coalition of volunteers can each make an equal voluntary contribution, while members whose incomes do not allow them to contribute at the same level contribute less.

4.1 N-symmetric equilibrium

In the utility function without budget constraint, there is no particular reason for any asymmetry in the amount of voluntary contributions. In the case of N-symmetric equilibrium, the voluntary contribution $d_i = d$ $\forall i \in N$ is equivalent to a voluntary increase in dues, raising them from $c$ to $c' = c + d$.

**Proposition 4** Without budget constraint, there exists a unique N-symmetric equilibrium of the voluntary contribution $d^*$ whose value is increasing in $N$, $\theta$, and $\alpha$.
and decreasing in \( c \) and \( X \).

Proof is given in Appendix C.

**Example** Sign of \( d^* \). Let us not restrict the sign for the voluntary contribution. A negative contribution is therefore equivalent to a reduction in dues. A positive value is equivalent to an increase in dues. To illustrate this, take a case where \( \theta = 1, \gamma = 0, \) and \( \alpha = 0.5 \). Replacing these parameters in (C.3), we study the conditions making \( d^* \) equal to zero:

\[
d^* = \frac{1}{4}N - c - \frac{X}{N} = 0. \tag{6}
\]

The solution of (6) gives a positive root \( N_+ \) and a negative root \( N_- \). \( d^* \) is negative between the two roots, so \( \forall N \leq N_+ \) and positive beyond.

Consequently, for

\[
N \leq 2(c + \sqrt{c^2 + X})
\]

\( d \leq 0 \), which is equivalent to a decrease in dues, and for

\[
N \geq 2(c + \sqrt{c^2 + X}),
\]

\( d \geq 0 \), which is equivalent to an increase in dues.

The considerations above concern the choice of the size of the association. Members may have incentives to recruit new members rather than reducing their dues, which might also mean reducing the scope of the collective action. This may be the case for the organizations Olson wrote about, which almost always welcome newcomers (Olson, 1971, p. 59.) Depending on the level of dues and on the structure of distribution of agents’ incomes, it may be more or less difficult to attract new members. Furthermore, when there are criteria for joining an association, this may limit its accessibility to potential members. Gordon and Babchuk (1959), who studied these membership criteria, identified the criteria of merit (as in the American So-
ciological Society, for instance) and the criteria of attributes, such as gender (e.g. feminist associations), origins or culture (for instance an association of Ukrainians), or simply certain social links between members.

Some associations may, on the contrary, prefer to restrict entry, like the so-called "status organizations" (Hansmann, 1986), which grant their participants a certain social status (Gordon and Babchuk, 1959.) In these organizations, the decision of an individual to join depends not only on the characteristics and the price of the collective good provided, but also on the characteristics of the members. This kind of association establishes a minimal status level (in social, economic or other terms) in addition to membership dues. When membership in an organization provides its members with social status, members may decide to limit the number of newcomers. This is the case described by Olson (1971):

"If the top "400" were to become the top "4000", the benefits to the entrants would be offset by the losses of old members, who would have traded an exalted social connection for one that might be only respectable" (Olson, 1971, p. 37.)

In any case, there exists a trade-off between the size of the organization and the level of dues.

4.2Coalitions and K-Symmetric Equilibrium

As we have already noted, one can consider a coordinated initiative of a group of $K$ members deciding collectively to make a symmetric voluntary contribution. Generally, such coordination is difficult to achieve and the organization costs can be high. However, as Olson (1971) noted, in some cases an existing group which has already met the organization costs may facilitate further collective action. More-
over, he noted that the aptitude of a group to provide a collective good can be partly explained by its origins and the factors that sustain it.

The idea that pre-existing organizations can facilitate the collective action has also been suggested by Coleman (1988) in the form of the concept of social capital. Groups of individuals who desire to contribute more can have various different origins. Besides a pre-existing organization, it may be founded on kinship links or shared social and cultural characteristics.

**Proposition 5** For any $K \leq N$, there exists a $K$-symmetric unique equilibrium such that $d_i = d^K \forall i \in K$, and $d_j = 0 \forall j \in N - K$. Moreover, $d^K$ is increasing in $K$, $\theta$, and $\alpha$ and decreasing in $N$, $X$, and $c$.

The proof is given in Appendix C.

**Example** Sign of $d^K$: Let us take an example of association characterized by the parameters $\theta = 1$, $\gamma = 0$, and $\alpha = 0.5$. Substituting them in (C.6) and resolving yields two solutions:

$$K_+ = 2\sqrt{Nc + X}$$
$$K_- = -2\sqrt{Nc + X}.$$

If $K \leq K_+$, then $d \leq 0$, and if $K > K_+$, then $d > 0$.

$K_+$ is interpreted here as a minimal size of $K$ positive equilibrium. In other words, it is the minimal size of an internal coalition formed by members to make voluntary contributions within the limits of their budget constraint.

If, nevertheless $K_+ > N$, we return to a $N$-symmetric equilibrium with $d^* < 0$ corresponding to a necessary reduction of compulsory payments.

The propositions 4 and 5 postulate, respectively, that at $N$-symmetric and $K$-symmetric
equilibria, the voluntary contribution decreases with an increase in the external resources $X$. This result illustrates the effect of crowding out currently highlighted by the models of "pure altruism" (Andreoni, 1988), where individuals are interested in total provision of the public good. As has been shown in a number of theoretical studies, under the "purely altruistic preferences" assumption, voluntary contributions are completely crowded out by public subsidies (Andreoni, 1988, 1990, Ribar and Wilhelm, 2002). However, this assumption has been challenged by empirical facts, notably in the nonprofit area (Steinberg, 1987, Andreoni, 1988, 1990, Ribar and Wilhelm, 2002). The assumption of "impurely altruistic preferences" (Andreoni, 1988), according to which individuals derive utility not only from total provision of the public good, but also from the act of giving, accounts better for the persistence of voluntary contributions in the presence of public financial support. Under this assumption, the crowding-out of voluntary contributions is incomplete (see for instance Steinberg, 1987.) Moreover, Rose-Ackerman (1986) shows the possible crowding-in effect of government grants in situations where voluntary contributions and grants are not perfect substitutes. However, in her model, crowding-in effects are only possible under the condition that the government is able to impose its rules on the organizations it supports concerning the services they provide.

According to the proposition above, the $K$ agents forming an internal coalition can contribute an equal amount, at the equilibrium, while others pay only the compulsory dues. The voluntary contribution decreasing in $N$ sheds light on the character of the sharing between the members of the coalition and the rest of the group. The larger the organization, the heavier the charge supported by the $K$ volunteers. Moreover, individuals may be averse to sharing a common good among a large number of people (Alesina and La Ferrara, 2000, Bergstrom and Goodman, 1973.) This aversion could result from the homophile preferences often used to explain
the formation of homogeneous groups of individuals with common features (Cohen, 1977.)

Finally, the proposition that voluntary contributions are increasing in $\theta$ describes the positive effects of the attractiveness of the public good on individual incentives to contribute. This theoretical finding is supported by a number of experimental studies, where an increase in the parameter of attractiveness leads to an increase in the mean of voluntary contributions (Hichri, 2004, p. 184.)

4.3 Budget constraint and partial K-symmetric equilibrium

What determines the emergence of homogeneous groups, besides homophily? As the sociological literature stresses, the size and composition of the population can also lead to the formation of homogeneous groups. For instance, McPherson and Lovin (1987) studied the influence of the size and diversity of voluntary associations on the formation of homophile links between members. They showed that homophily is not merely a natural tendency of individuals, but that the social structure can also favor or constrain individual choices. Therefore, among the factors influencing the internal groups in associations, we should distinguish between constrained and freely-chosen homophily.

We shall incorporate this idea into our model in the last section of this chapter, by introducing individual preferences for the subjective quality of the public good. Here, we introduce some heterogeneity by imposing the budget constraint that influences agents’ choices. In the case with a budget constraint, and with N-symmetric equilibrium, it is possible that

$$\exists i \in N : w_i - c - d^* < 0.$$ 

Now, some members of the association cannot contribute at the optimal level $d^*$ and their contributions cannot exceed the level of $\bar{d} = w_i - c$ limited by their bud-
Proposition 6  Given a distribution of revenues, there exists a unique value of \( K \leq N \) and a unique partially \( K \)-symmetric equilibrium, such that \( d_i = d^K \quad \forall i \in K = \{k \mid w_k - c - d^K \geq 0\} \) et \( d_j = w_j - c \quad \forall j \in (N - K) \).

The proof is given in Appendix C.

5 Non-symmetric equilibria

In this section, we study the relation between the level of voluntary contributions and incomes. This relation is founded on the non-constant character of the marginal contribution of a monetary unit to the individual utility. Taking the individual utility model based on division of the budget between private consumption and contributions to the public good, we can consider that the net income of the agent \( i \), or \( w_i - c - d_i \) contributes to her utility in a concave way. Consequently, we can note:

\[
U_i = (w_i - c - d_i)^\beta + \frac{\theta}{N^\gamma}(Nc + \sum_{j=1}^{N} d_j + X)^\alpha.
\]

(7)

5.1 Equilibrium without budget constraint

Proposition 7  Without budget constraint, there exists a unique equilibrium, in which any agent \( i \) fixes her voluntary contribution in such a way that her net income is brought to the level \( w_i - c - d_i = w_0 \), where \( w_0 > 0 \) is a reference level common to all the agents.

The proof is given in Appendix D.
**Example** Let us consider a special case: if $\alpha = \beta$, then

$$w_0 = \frac{\Omega + X}{N + \frac{1}{N^\theta}}. \quad (8)$$

The proposition 7 shows how the agents chose their level of voluntary contribution at a non-symmetric equilibrium. In fact, in this case, each agent pays a personalized price for the public good. The ability of voluntary organizations to establish personalized prices (or a system of price discrimination) has already been stressed in the economic literature (Hansmann, 1981, Ben-Ner, 1986.) According to Ben-Ner, the non-market interactions between members, (kinship links, neighborhood, common background), as well as the non-profit distribution constraint, allow these organizations to reveal demands and to establish personalized prices.

### 5.2 Budget constraints

We now introduce budget constraints: $\forall i \in N, d_i \geq 0$, and $d_i \leq w_i - c$. In a solution under constraint $d_i < 0$ becomes $d_i = 0$, while $d_i > w_i - c$ becomes $d_i = w_i - c$.

Nevertheless, $d_i > w_i - c \iff w_i - c - w_0 > w_i - c$, and thus $\iff w_0 < 0$, which contradicts the previous proposition.

Thus, the constraint $d_i \geq 0$ still remains.

Here, $d_i < 0$ if and only if $w_i < w_0 + c$, and the set of agents with incomes lower than $w_0 + c$ do not bring voluntary contributions.

**Proposition 8** There exists a unique equilibrium under constraint, in which the richest agents of a sub-group $K \leq N$ contribute at such a level that their net incomes are reduced to a common reference level $w_i - c - d_i = w_0 > 0$ for all the agents in
$K$, while the agents from $N - K$, whose incomes are less than $w_0 + c$, do not make any voluntary contributions.

For proof see Appendix D.

**Example** As a special case, take $\alpha = \beta$. Then

$$w_0 = \frac{\Omega_K + X + (N - K)c}{K + \left(\frac{\theta}{\gamma r}\right)^{\frac{1}{1-\alpha}}}. \tag{9}$$

**Remark** The result where all the agents are reduced to the same level $w_0$ of net income is somewhat simplistic. This is due to the fact that in this model, the share of the initial allocation dedicated to private consumption corresponds to an homogeneous good with a unique technology of consumption. However, in real life, disposable income is rather devoted partly to differentiated baskets of goods characterized by diversified consumption technologies and partly to savings.

### 6 Volunteering and subjective quality

Now, let us assume that the agents can contribute to the public good not only money, but also their time and effort. One can differentiate three main types of theoretical framework for explaining volunteering: pure public good models, private consumption models and investment models (Ziemek, 2006.) Pure public good models consider that volunteers are motivated exclusively by total provision of the public good. Private consumption models focus on personal rewards, like social status or simply a warm glow. Investment models may be considered as a special case of private consumption model, as volunteering provides certain benefits associated with the accumulation of human and social capital.

The model we describe below is situated between the "pure public goods" and
“private consumption” approaches, as the act of volunteering is not independent of the characteristics of the public good to which it contributes. We consider that volunteering provides an additional utility to the one derived from total provision, as in Steinberg (1987), for instance. Steinberg’s approach, considered as a "mixed public-private good approach", represents a case of an "impurely altruistic model" (Andreoni, 1990). The individual utility function includes the public goods provided through the agent’s individual voluntary contribution, public resources and the contributions of others. The individual contribution and the other agents’ contributions are complementary when the individual is faced with social comparisons within her group of reference. When the individual is motivated only by total provision, her contributions and the contributions of others are perfect substitutes.

In our model, each member $i$ can contribute to the public good by making a non-monetary contribution of effort noted $v_i$, which is subjective value. This effort allows agents to influence the quality of the public good according to their own preferences. The quality is subjective insofar as it cannot always be evaluated objectively. In other words, the quality of a good (or a service) is subjective when it is evaluated through individual perceptions rather than standardized measures.

One notable example of the use of subjective quality comes from the field of child day-care. Here, the measurement of quality is influenced by parents’ preferences concerning certain service characteristics, making it more subjective than the standards elaborated by experts. For instance, it may be important to know whether the provider of the child-care service speaks the same language, shares the same values or the same religion. This implies that an individual’s perception of quality may differ not only from others’ perceptions, but also from the more objective evaluations made by experts using an index of quality (Farquhar, 1993, Moss, 1994.) As Evers and Riedel (2004) have shown in a German case study, the development of non-profit child-care centers was mainly driven by the desire of some parents to create
a service in keeping with certain pedagogical, ideological or religious principles. Moreover, as Kushman (1979) observed in the United States, nonprofit child-care centers tend to provide more labor-intensive services, sometimes including parent participation, than private ones.

We therefore suggest that the members of an association can volunteer in the hope of influencing the subjective quality of the good or service being provided. In the field of child care, for instance, the role of volunteers can consist in organizing activities according to their cultural background or pedagogical convictions. In the field of the arts, active members can influence programming choices according to their personal tastes. Finally, in a charity helping the poor, the more active volunteers can influence the criteria of eligibility or the order of priority (Rose-Ackerman, 1986.)

Two aspects must be taken into consideration here:

- the cost of the effort.

By convention we measure effort $v_i$ as a fraction of the individual’s disposable time that is dedicated to the volunteering, or $v_i \in [0, 1]$. For $i$, the effort $v_i$ represents an opportunity cost that is proportional to her income. We assume that each agent $i$ works a length of time 1 to earn an income $w_i$. Thus, the opportunity cost can be written as $v_i w_i$.

- the impact on the subjective value of the good.

The subjective value of the public good for the agent $i$ depends both on the total amount of effort provided by all the volunteers and on the weight of her own effort as a proportion of the total. The combination of both can differ according to how ”individualistic” the association is. So agents can attach more or less value to the public good itself, but they also value their own contributions more or less highly.
Thus, the subjective level of effort can be written as follows:

\[(1 - \delta) \sum_{j=1}^{N} v_j + \delta v_i = (1 - \delta)v_{-i} + v_i.\] (10)

where $\delta$ is the parameter of "individualism". Two extreme cases can be considered:

- $\delta = 0$ leads to $\sum_{j=1}^{N} v_j$ and expresses a purely collective orientation. In other words, the effort of a volunteer is added to the sum of efforts provided by other agents;
- $\delta = 1$ leads to $v_i$ and designates a purely individualistic orientation, where the personal contribution is the only one valued, in spite of the effort provided by others.

We consider that $\delta$, the so-called parameter of individualism, characterizes the type of association. Under some conditions, it is possible to link this parameter with the typology of voluntary associations (see for instance Gordon and Babchuk, 1959), defining them as "expressive" or "instrumental groups". The main objective of the first type of association is to provide common activities for their members. The members are direct beneficiaries of organized activities (for instance a country-club.) A $\delta$ equal to one denotes a purely expressive association, inasmuch as participation in its activities provides a direct gratification for the members. On the contrary, the main function of so-called instrumental associations is directed outside the organization. This kind of association aims to create and maintain a normative condition or a commitment (Gordon and Babchuk, p. 25) and exercises a social influence. Examples include associations for the defence of the rights of minorities and the poor, or certain political and religious organizations. The extreme case of such an association is denoted by a value of $\delta$ equal to zero. In other words, it is assumed that the members are driven by ideological principles or common values,
rather than the pleasure of participating in a shared activity. In real life, voluntary associations can accomplish both functions. For example, some associations may have an expressive function at the local level and an instrumental function at the national level. Intermediate associations may be situated between the two extreme cases with a $\delta$ more or less close to 0 or to 1.

In our model, we introduce effort into the production function of the public good by means of a Cobb-Douglas function. The utility of the agent $i$ is now written as:

$$U_i = w_i - c - d_i - w_i v_i + \frac{\theta}{N^\gamma} (Nc + \sum_{j=1}^{N} d_j + X)^\alpha ((1 - \delta) \sum_{j=1}^{N} v_j + \delta v_i)^\beta$$

(11)

with $\alpha$ and $\beta \in ]0,1[.$

In a standard way, each agent $i$ determines her pecuniary contribution $d_i$ and her effort $v_i$ so as to maximize her subjective utility $U_i$, according to her rational expectations about the levels of voluntary contributions $\sum_{j \neq i} d_j = d_{-j}$ and effort $\sum_{j \neq i} v_i = v_{-i}$ made by the other agents.

**Proposition 9** At equilibrium, each agent fixes her levels of voluntary contribution and effort $(d_i, v_i)$ by attributing to the latter a relative weight depending on her income. The lower the income, the higher the relative weight of the effort. In other words, a poorer agent will compensate for her weak monetary contribution by a greater effort. Moreover, for all the agents, the level of direct effort is an increasing function of the degree of individualism $\delta$.

For proof see Appendix E.

The model above shows that pecuniary contributions and volunteering can well be made jointly. This conforms to the stylized facts, whereby monetary donations and volunteering often come together. Generally, the explanation proposed for this
phenomenon is that donors need to control the good use of their donations. In this paper, we put forward the idea that the trade-off between monetary contributions and volunteering can be based on the desire to influence the quality of the public output. Moreover, the valuation of the public good becomes socially-based, to the extent that it depends on the agents’ characteristics (culture, education, ideological considerations, etc.)

Here, the concept of service assumes particular importance. The outputs of non-profit organizations are mostly represented by services (Weisbrod, 1977, Hansmann, 1980). It can be difficult to evaluate a service when it has both immediate and long-term impacts, or when the perceptions of its quality differ across individuals. The diversity of criteria by which a service can be appraised influences the possible trade-off between giving and volunteering. Pecuniary contributions and volunteering may well influence the characteristics of the service differently. Thus, agents can contribute both money and effort. Another interesting issue highlighted in the model is the level of effort provided by an agent as a function increasing in $\delta$, the parameter of individualism characterizing the association. In other words, the model predicts that volunteering is likely to be more important in expressive associations (Gordon and Babchuk, 1959), i.e. those essentially oriented towards the interests of their members. One conclusion we can draw from this result is that to attract a greater effort of volunteering, an association should take into account the personal interests of its volunteers. This can be achieved through awards, such as public recognition and social status, but also through socialization, dialogue and communication (Laville and Sainsaulieu, 1997, p. 292).

This result is in keeping with some stylized facts about volunteering at the international level in general, and in the United States and France in particular. At the international level, Salamon et al. (2003) highlight the important role of vol-
unteers in the field of culture and arts, accounting for a quarter of total volunteering (p. 24). According to Schervish and Havens (1997), volunteers in the United States largely benefit from the activities they create. Finally, in France, more than half of the members of expressive associations (e.g. shared leisure activities, social clubs) participate at least once a week in the shared activities as organizers, while this figure is only 34 per cent in the instrumental associations, like those for the defence of minority interests. Moreover, nearly half of the time dedicated to volunteering is mobilized in the areas of sport, culture and leisure, which are the biggest consumers of this input (Prouteau and Wolff, 2004.) The results of Prouteau and Wolff (2004) confirm the hypothesis that in France, at least, volunteering by adults is often reinforced by specific needs for services (children’s education, for instance). In an empirical study using U.S. data on married women’s voluntary labor, Carlin (2001) finds that participation rates are positively related to the number of children. As Ziemek (2006) shows in a cross-country study, the presence of young children in a household positively influences the egoism and investment motivation of volunteers.

As regards the effect of income on volunteering, according to (E.3), the level of effort is a decreasing function of \( w \). In other words, the effort of volunteering is likely to rise when the wage rate decreases. As empirical support to this finding, we can cite the inverse relation between voluntary labor and wage rates documented by Menchik and Weisbrod (1987). In addition, Andreoni et al. (1996) find a relatively substantial negative effect of the net wage on volunteer hours. However, the effects of wage rate on volunteering may be sensitive to the type of volunteers’ motivations or other determinants.
7 Conclusion

Collective-consumption goods can be provided by voluntary associations. In this paper, we consider an important feature of these organizations, concerning the issue of personal commitment to a common action. We develop a model of voluntary association of individuals who are interested in obtaining advantages from the provision of a common good. At the first step, we consider a set of agents who benefit from a quantity of a good in a uniform manner. We show the existence of several symmetric and non-symmetric equilibria corresponding to the different modes of emergence of associations identified in the sociological literature (Laville and Sainsaulieu, 1997). At the N-symmetric equilibrium corresponding to the case of a "for-self" association, all the agents are equal in the sense that they all make a fixed voluntary contribution. Voluntary contributions are crowded out by compulsory payments and by exogenous resources. At the K-symmetric equilibrium, where one can distinguish between two categories of agents, the "strong" and the "weak" ones, only $K$ agents make voluntary contributions. This case corresponds to the case of an association created "for others". Here, the effects of crowding out are also present. Moreover, the level of voluntary contributions diminishes with the size of the organization. Both at the N-symmetric and the K-symmetric equilibria, the amount of voluntary contributions grows with the attractiveness of the public good.

Then, by introducing a budget constraint, we show the existence of a unique partial K-symmetric equilibrium, in which the $K$ richest agents give an equal amount, while the others give the rest of their disposable income after dues. When it is assumed that the net income of the agent enters the utility function in a concave way, we find that there exists a non-symmetric equilibrium at which all the agents fix their voluntary contributions in such a way as to make their net disposable in-
comes equal. Finally, in the same configuration and after re-introducing the budget constraint, we show the existence of a unique equilibrium at which only the richest members form an internal coalition to make additional voluntary contributions. The members of this group inside the association give an amount that makes their net incomes equal.

At the second step, we introduce the quality of the public good into the basic model. Here, quality is considered simultaneously as a public and private component of the good provided, while quantity is generally considered as the only feature of the public good. An exception is provided by Ben-Ner (1986), who considers quality as a public component and quantity as a private component, in the sense that consumers only buy the quantity they demand. The model presented above is reinforced by a number of works in the empirical literature concerning the positive relationship between voluntary contributions and the attractiveness of the public good, incomplete crowding-out phenomena, the importance of the organization size and its effects on voluntary contributions, and the positive link between incomes and the level of voluntary contributions. Moreover, according to the stylized facts, the effort of volunteering is more important in the organizations oriented towards the direct interest of the members. Thus, the rewards of volunteers are not pecuniary. Rather, the model proposed above stresses how dialogue, social acknowledgment, and what some call "accepted differentiation" (Laville and Sainsaulieu, 1997) are important. Even if most of the results of the model essentially correspond to the stylized facts, they can depend on the structure of the model. An important assumption made in the model is that individuals may desire not only to increase production of the public good, but also to influence its quality by volunteering. However, this does not exclude other possible incentives of volunteers, which are not taken into account in the model. Moreover, the incentives of volunteers to devote time and effort may differ, depending on the people and the social context. The degrees of motivation
of individuals can also be heterogeneous. This heterogeneity of incentives could be an interesting issue for future works.

Finally, the main contribution of this paper consists in highlighting voluntary association as a group of individuals formed around a common intention. But this does not exclude that possibility that the members of an association may have their own aspirations and conceptions of the good or service they want to provide. This idea is formalized by introducing the concept of subjective quality, according to which the perceptions of the quality of a good (or service) may well differ across persons. Accepting these differences, what has been called "accepted differentiation", allows the actors to promote a collective action. As Laville and Sainsaulieu (1997) observed, associative activity is an exercise in social cohesion; it does not exclude the expression of differences that come together around a shared project. In other words, as the saying goes, "people can share the same bed without sharing the same dreams."

Acknowledgments

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A Proofs of propositions 1 and 2

Let us assume first symmetric voluntary contributions, in other words \( d_i = d \) for each \( i \), and note \( c' = c + d \). Now, the equation (2) can be re-written as

\[
f(N) = \frac{1}{N^{\gamma}} (Nc' + X)^\alpha, \tag{A.1}
\]

To make \( f(N) \) defined and continuous, let us assume that

\[
N c' + X \geq 0,
\]

or that

\[
N \geq -\frac{X}{c'} = N_0. \tag{A.2}
\]

When \( X < 0 \) (namely, in the presence of fixed costs), this condition means that the number of participants must be high enough for the sum of individual contributions \( c' \) to compensate the fixed cost \( X \).

When \( X \geq 0 \), this condition holds for any value of \( N \geq 0 \).

The derivative of the production function of the public good (2) can be written as follows:

\[
\frac{df}{dN} = \frac{Nc'(\alpha - \gamma) - \gamma X}{N^{\gamma+1}(Nc'+X)^{1-\alpha}}, \tag{A.3}
\]

Given (A.2), the sign of the derivative will be the one of the numerator (A.3), or

\[
sg \frac{df}{dN} = sg(Nc'(\alpha - \gamma) - \gamma X). \tag{A.4}
\]
(i.) When $X \geq 0$, namely in the presence of public subsidies,

- $\gamma \geq \alpha$

$$\frac{df}{dN} \leq 0, \forall N. \quad (A.5)$$

In this case, association does not make sense as the sharing rule described by $\gamma$ cancels the gain of individual utility.

- $\gamma < \alpha$

The variation of the sign of the derivative is given in the table below.

<table>
<thead>
<tr>
<th>$N$</th>
<th>$N_1$</th>
<th>$\frac{df}{dN}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$-\frac{\gamma X}{(\alpha - \gamma)c^2}$</td>
<td>$&lt; 0$</td>
</tr>
</tbody>
</table>

Beyond $N_1$, each additional contribution leads to a gain in individual utility. There is no overcrowding, but there exists a minimal size of association which is necessary for association to make sense.

(ii.) When $X < 0$, namely in the presence of fixed costs, two cases need to be studied:

- If $\gamma \leq \alpha$, the production function is increasing with the number of contributors if their number is higher than $N_0$. In this case, there is no overcrowding.

$$Nc'(\alpha - \gamma) - \gamma X > 0, \forall N. \quad (A.6)$$

Thus,

$$\frac{df}{dN} \geq 0, \forall N \geq \frac{-X}{c} = N_0. \quad (A.7)$$
• If $\gamma > \alpha$,

\[ Nc'(\alpha - \gamma) - \gamma X = 0 \]  

(A.8)

for a number of participants $N$ equal to

\[ N_1 = \frac{-\gamma X}{(\gamma - \alpha)c'} \]  

(A.9)

the table of variations of the derivative will be the following:

Table A.2
Variations of the production function with $N$ in the presence of fixed costs

<table>
<thead>
<tr>
<th>$N$</th>
<th>$N_1 = -\frac{\gamma X}{(\gamma - \alpha)c'}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{df}{dN}$</td>
<td>$&gt; 0$</td>
</tr>
</tbody>
</table>

Beyond $N_1$, the production function generates overcrowding. In other words, when the number of participants is higher than $N_1$, the amount of good produced diminishes with the number of members.

Note that $N_1 > N_0$. The number of members $N_0$ represents a minimal size of association. Beyond this size and up to $N_1$, effects of overcrowding are absent, namely the production of the public good increases with the number of members.

\[ \square \]

A.1 Extension 1

Now let us suppose that $K$ first members, founders for instance, have fixed the level of the voluntary contribution, while the newcomers contribute only the compulsory amount $c$. For each $N \in [N_0, K]$, the previous results hold.
Beyond this threshold, $\forall N > K$ the production function is written as follows:

$$f(N) = \frac{1}{N^\gamma} (Nc + Kd + X)^\alpha. \tag{A.10}$$

This situation is equivalent to the previous case with $c' = c$ and

$$Y = X + Kd. \tag{A.11}$$

Thus, the previous results are found with

$$N_0 = -\frac{Y}{c}$$

and

$$N_1 = \frac{\gamma Y}{(\alpha - \gamma)c}. \tag{A.12}$$

An important issue here is to situate $N_1$ with regard to $K$.

If $Y \geq 0$ and $\gamma < \alpha$,

$$N_1 = \frac{\gamma Y}{(\alpha - \gamma)c} \geq K$$

if and only if

$$\gamma Y \geq (\alpha - \gamma)cK. \tag{A.13}$$

By substituting (A.11) in (A.13) we obtain

$$K[c(\alpha - \gamma) - d] \leq \gamma X. \tag{A.14}$$

- If the term of the equation (A.14) $c(\alpha - \gamma) - d > 0$, i.e. if the compulsory amount is high enough relative to voluntary contributions, then we obtain

$$K < \frac{\gamma X}{c(\alpha - \gamma) - d} = s. \tag{A.15}$$
Therefore we obtain the following table of variations, where two alternative cases depend on the relative position of $K$ with regard to $s$.

<table>
<thead>
<tr>
<th>$N_0$</th>
<th>$s$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$d,\frac{f}{dN}$</td>
<td>$&lt; 0$</td>
</tr>
<tr>
<td>$K$</td>
<td>$N_1$</td>
</tr>
</tbody>
</table>

Thus, for a population that is large enough, there is no overcrowding.

- In the case where $c(\alpha - \gamma) - d \leq 0$ and $X \geq 0$, the inequality (A.14) holds for any $K \geq 0$ (here we are in the alternative case $K \leq N_1$).

  If $X < 0$, we find the two alternative cases above.

  Finally, if $Y < 0$, we find the previous results.

### A.2 Extension 2

If beyond the number $K$ it is possible to make voluntary contributions, the situation of every member is improved. Overcrowding remains absent, but it can occur when $Y < 0$ and $\gamma > \alpha$, beyond a threshold possibly moved forward.

### A.3 Extension 3

If below $K$, the voluntary contributions vary, one can substitute $Kd$ by $D = \sum_{j=1}^{K} d_j$. The result still holds.
B Proofs of proposition 3 and corollary 3.1

B.1 Proof of proposition 3

The utility for the member $N$ is written as

$$ U_N = w_N - c - d_N + \frac{\theta}{N^\gamma} (Y_N + d_N)^\alpha, \quad \text{(B.1)} $$

where

$$ Y_N = Nc + \sum_{j=1}^{N-1} d_j + X. \quad \text{(B.2)} $$

$N$ has an incentive to contribute when

$$ \frac{\partial U_N}{\partial d_N} \geq 0. \quad \text{(B.3)} $$

$$ \frac{\partial U_N}{\partial d_N} = -1 + \frac{\theta \alpha}{N^\gamma} (Y_N + d_N)^{\alpha-1}. \quad \text{(B.4)} $$

Thus, we obtain the following condition:

$$ \theta \alpha \geq N^\gamma (Y_N + d_N)^{1-\alpha}. \quad \text{(B.5)} $$

Therefore, $N$ will have incentives to contribute a positive amount if and only if

$$ \theta \alpha \geq N^\gamma (Nc + \sum_{j=1}^{N-1} d_j + X)^{1-\alpha}. \quad \text{(B.6)} $$

The right-hand term is increasing in $N$.

Then, formation of the internal group is studied in the situation of symmetry. In other words, here we analyze the situation where a group of individuals in an association each decide to provide an equal amount. This is equivalent to the decision
to raise the compulsory payment. Now, the initial individual utility is written as

\[ U^G_0 = w_i - c + \frac{\theta}{N^{1 - \gamma}} (Nc + X)\alpha, \quad (B.7) \]

We study the derivative with regard to \( c \):

\[ \frac{\partial U}{\partial c} = -1 + \theta \alpha N^{1 - \gamma} (Nc + X)^{-1}. \quad (B.8) \]

Consequently, \( \frac{\partial U}{\partial c} \geq 0 \) if and only if

\[ \theta \alpha \geq \frac{(Nc + X)^{1 - \alpha}}{N^{1 - \gamma}}. \quad (B.9) \]

We study the variations of the right-hand side term of (4):

\[ f(N) = \frac{(Nc + X)^{1 - \alpha}}{N^{1 - \gamma}} \quad (B.10) \]

\[ \frac{\partial f}{\partial N} = \frac{(Nc + X)^{1 - \alpha} N^{-\gamma}}{N^{2(1 - \gamma)}} (\gamma Nc - (1 - \gamma)X). \quad (B.11) \]

Consequently,

\[ \text{sg} \frac{\partial f}{\partial N} = \text{sg}(\gamma Nc - (1 - \gamma)X), \quad (B.12) \]

and thus, \( \frac{\partial f}{\partial N} \geq 0 \) if and only if

\[ N \geq \frac{1 - \gamma X}{\gamma c}. \quad (B.13) \]

\[ \square \]
B.2 Proof of corollary 3.1

The level $c^*$ is obtained from (B.8) as solution of $\frac{\partial U}{\partial c} = 0$. It can be written as:

$$c^* = \frac{(\theta \alpha)^{\frac{1}{1-\alpha}} N^{\frac{1-\gamma}{1-\alpha}} - X}{N}. \quad (B.14)$$

Now the variation of $c^*$ as a function of $N$ can be studied:

$$\frac{\partial c^*}{\partial N} = \frac{(\theta \alpha)^{\frac{1}{1-\alpha}} N^{\frac{1-\gamma}{1-\alpha}} - X - \alpha N^{\frac{1-\gamma}{1-\alpha}} + X}{N^2}. \quad (B.15)$$

Given the assumption $X \geq 0$ et $\alpha > \gamma$,

$$\frac{\partial c^*}{\partial N} > 0.$$

\[\square\]

C Proofs of propositions 4, 5, and 6

C.1 Proof of proposition 4

The utility function in N-symmetric equilibrium can be written as

$$U_i = w_i - c - d + \frac{\theta}{N^\gamma} (N(c + d) + X)^\alpha, \quad (C.1)$$

$$\frac{\partial U_i}{\partial d} = -1 + \theta \alpha N^{1-\gamma} (N(c + d) + X)^{\alpha-1}. \quad (C.2)$$
By assuming the right-hand term of (C.2) equal to zero, we obtain the optimal value of voluntary contribution in N-symmetric equilibrium without budget constraint:\footnote{The second order condition holds if \(X \geq 0\) and \(\alpha < 1\).}

\[
d^* = (\theta \alpha)^{1/\alpha} N^{\alpha - \gamma} N^{1-\alpha} - c - \frac{X}{N}.
\] (C.3)

From equation (C.3) follows that \(d^*\) is increasing in \(\theta\).

Let us note

\[
d^* = (\lambda \alpha) N^{1-\gamma} \frac{1}{N} - c - \frac{X}{N},
\]

with

\[
\lambda = \theta N^{1-\gamma}.
\] (C.4)

We study the function \(f(\alpha) = (\lambda \alpha)^{1/\alpha}\) avec \(\alpha < 1\).

\[
\frac{df}{d\alpha} = \frac{\lambda}{1-\alpha} (\lambda \alpha)^{\alpha-1/\alpha}.
\]

\(\frac{df}{d\alpha} > 0\), \(d^*\) is therefore increasing in \(\alpha\).

We now study the variation of \(d^*\) with \(N\):

\[
\frac{\partial d^*}{\partial N} = (\theta \alpha)^{1/\alpha} \frac{\alpha - \gamma}{1-\alpha} N^{\alpha - \gamma - 1} + \frac{X}{N^2} > 0.
\]

On the contrary, it follows immediately from (C.3) that \(d^*\) is decreasing in \(c\) and in \(X\). □

C.2 Proof of proposition 5

At the K-symmetric equilibrium, the utility function is written as follows:

\[
\forall i \leq K \quad U_i = w_i - c - d + \frac{\theta}{N^\gamma} (Nc + Kd + X)^\alpha,
\] (C.5)
Maximizing (C.5) yields

\[ d^K = \left( \frac{\theta a K}{N^q} \right)^{\frac{1}{1-a}} - Nc - X \].

(C.6)

From (C.6) it follows that \( d^K \) is increasing in \( \theta, \alpha, \) and \( K \) and decreasing in \( N, X, \) and \( c. \)

\[ \square \]

\[ \]

C.3 Proof of proposition 6

Let us range the population of agents in the decreasing order of incomes as follows

\[ j > i \Rightarrow w_j \leq w_i. \]

The result is a straightforward consequence of the properties shown by Foray, Thoron, and Zimmermann (2007) applied to the previous model of equilibrium with constraints. The distribution of voluntary contributions is presented by the figure (C.1).

Fig. C.1. The distribution of voluntary contributions
Now, the individual utility is written for any $i \in K$ as follows:

$$U^K_i = w_i - c - d^K + \frac{\theta}{N^\gamma}(Nc + Kd^K + D^{N-K} + X)^\alpha, \quad (C.7)$$

where

$$D^{N-K} = \sum_{j> K} w_j - c.$$

or

$$U^K_i = w_i - c - d^K + \frac{\theta}{N^\gamma}(K(c + d^K) + \Omega^{N-K} + X)^\alpha, \quad (C.8)$$

where

$$\Omega^{N-K} = \sum_{j \in (N-K)} w_j.$$

For any $i \in K$,

$$\frac{\partial U_i}{\partial d^K} = -1 + \frac{\theta \alpha K}{N^\gamma} (K(c + d^K) + \Omega^{N-K} + X)^{\alpha-1}. \quad (C.9)$$

By assuming $\frac{\partial U_i}{\partial d^K} = 0$, we obtain an optimal value of contributions for any $i \in K$:

$$d^K = \left[ \left( \frac{\theta \alpha K}{N^\gamma} \right)^\frac{1}{\alpha - 1} - \Omega^{N-K} - X \right]^{\frac{1}{\alpha - 1}} K - c. \quad (C.10)$$

According to Thoron and Zimmermann (2007), the value of $K$ is adjusted in such a way, that $w_k - c \geq d^K$ and $w_{K+1} - c < d^K$.  □
D Proofs of propositions 7 and 8

D.1 Proof of proposition 7

Now, the first order condition is written as

\[
\frac{\partial U_i}{\partial d_i} = -\frac{\beta}{(w_i - c - d_i)^{1-\beta}} + \frac{\theta}{N^\gamma (Nc + \sum_{j=1}^N d_j + X)^{1-\alpha}}. \tag{D.1}
\]

The second order condition becomes

\[
\frac{\partial^2 U_i}{\partial d_i^2} = -\frac{\beta (1 - \beta)}{(w_i - c - d_i)^{2-\beta}} - \frac{\theta \alpha (1 - \alpha)}{N^\gamma (Nc + \sum_{j=1}^N d_j + X)^{2-\alpha}} < 0. \tag{D.2}
\]

The non-constrained equilibrium is obtained by assuming \( \frac{\partial U_i}{\partial d_i} = 0 \) \( \forall i \in N \) under condition

\[w_i - c - d_i > 0. \tag{D.3}\]

We obtain therefore

\[
\frac{\beta}{(w_i - c - d_i)^{1-\beta}} = \frac{\theta}{N^\gamma (Nc + \sum_{j=1}^N d_j + X)^{1-\alpha}}. \tag{D.4}
\]

It follows consequently that \( \forall i \neq j \)

\[
\frac{\beta}{(w_i - c - d_i)^{1-\beta}} = \frac{\beta}{(w_j - c - d_j)^{1-\beta}}
\]

or that \( \forall i \in N, w_i - c - d_i = w_0 \), and thus

\[d_i = w_i - w_0 - c. \tag{D.5}\]
Thus, after voluntary contribution, all the agents are brought to the same level of net income $w_0$.

Now, we can write

$$\sum_{j=1}^{N} d_j = \sum_{j=1}^{N} w_j - N(c + w_0) = \Omega - N(c + w_0),$$

from which, by substituting in (D.4) we obtain

$$\frac{\beta}{w_0^{1-\beta}} = \frac{\theta}{N^\gamma} \frac{\alpha}{(\Omega + X - Nw_0)^{1-\alpha}} \quad (D.6)$$

It follows that $w_0$ solution of the implicit equation (D.6) is strictly positive, which satisfies the condition (D.3). □

\[D.2 \quad \text{Proof of proposition 8}\]

As in Foray, Thoron, and Zimmermann (2007), if the $N$ agents are ranged in a decreasing order of incomes, there exists a $K \leq N$ satisfying the following conditions:

The individual utility is written $\forall i \in K$

$$U_i = (w_i - c - d_i)^{\beta} + \frac{\theta}{N^\gamma} (Nc + \sum_{j=1}^{K} d_j + X)^{\alpha}. \quad (D.7)$$

In the same way as in the proposition 7, $\forall i \in K$

$$d_i = w_i - w_0 - c.$$

However, this time

$$\sum_{j=1}^{K} d_j = \sum_{j=1}^{K} w_j - K(c + w_0) = \Omega_K - K(c + w_0),$$
and consequently, $w_0$ is solution of the implicit equation

$$\frac{\beta}{w_0^{1-\beta}} = \frac{\theta}{N^\gamma} \frac{\alpha}{(\Omega K + X - Kw_0 + (N - K)c)^{1-\alpha}}.$$  

(D.8)

\[\square\]

E Proof of proposition 9

To simplify the expression of the utility function, let us note $w = w_i$, $m = d_i$, $M = Nc + X + d_{-i}$, $f = v_i$, $F = (1 - \delta)v_{-i}$, $z = \frac{\theta}{N^\gamma}$ et $x = w_i - c - d_i$. The latter represents the share of the budget of $i$ available for her private consumption.

Thus, the agent $i$ is solving the following:

$$\max U(x, m, f) = x - wf + z(M + m)^\alpha(F + f)^\beta$$

(E.1)

under a budget constraint

$$x + m + c - w = 0$$

(E.2)

Using the Kuhn and Tucker theorem and noting the multiplicator corresponding to the constraint (E.2) as $\lambda$, we write the marginal utilities and the respective first order conditions (the marginal utilities are proportional to the prices):

$$\frac{\partial U}{\partial x} = 1 = \lambda p_x = \lambda \quad (i)$$

$$\frac{\partial U}{\partial m} = \alpha z \frac{(F + f)^\beta}{(M + m)^{1-\alpha}} = \lambda p_m = \lambda \quad (ii),$$

and

$$\frac{\partial U}{\partial f} = \beta z \frac{(M + m)^\alpha}{(F + f)^{1-\beta}} - w = \lambda p_f = 0. \quad (iii)$$
From (i) it follows that $\lambda = 1$.

Consequently, the first order conditions (ii) and (iii) can be written as

$$\alpha z (F + f)^{\beta} = (M + m)^{1-\alpha} \quad (ii')$$

and

$$\beta z (M + m)^{\alpha} = w(F + f)^{1-\beta} \quad (iii')$$

By making a cross product of the terms of these equations, we obtain:

$$\alpha w (F + f) = \beta (M + m)$$

$$\iff f = \frac{\beta}{\alpha w} (M + m) - F. \quad (E.3)$$

□

References


Foray, D., Thoron, S., Zimmermann, J-B. 2007; Open Software: Knowledge Openness


